7. Values of Trigonometric Functions at Sum of Difference of Angles

Exercise 7.1

1. Question

If sinA = 4/5 And cosB = 5/13, where 0 <A, B < π/2, find the values of the following:

(i) sin(A +B)

(ii) cos(A +B)

(iii) sin(A –B)

(iv) cos(A -B)

Answer

Given sinA = 4/5 And cosB = 5/13

We know that $\cos A = \sqrt{1-\sin^2 A}$ and $\sin B = \sqrt{1-\cos^2 B}$ where 0 <A,B < π/2

$$\Rightarrow \cos A = \sqrt{1-\left(\frac{4}{5}\right)^2} \text{ and } \sin B = \sqrt{1-\left(\frac{5}{13}\right)^2}$$

$$\Rightarrow \cos A = \sqrt{1-\frac{16}{25}} \text{ and } \sin B = \sqrt{1-\frac{25}{169}}$$

$$\Rightarrow \cos A = \sqrt{\frac{9}{25}} \text{ and } \sin B = \sqrt{\frac{144}{169}}$$

$$\therefore \cos A = \frac{3}{5} \text{ and } \sin B = \frac{12}{13}$$

Then,

(i) Sin(A +B)

<u>We know that sin(A +B) = sinA cosB + cosA sinB</u>

$$= \frac{4}{5}\times\frac{5}{13}+\frac{3}{5}\times\frac{12}{13}$$

$$= \frac{20}{65}+\frac{36}{65}$$

$$= \frac{56}{65}$$

(ii) Cos(A +B)

<u>We know that cos(A +B) = cosA cosB - sinA sinB</u>

$$= \frac{3}{5}\times\frac{5}{13}-\frac{4}{5}\times\frac{12}{13}$$

$$= \frac{15}{65}-\frac{48}{65}$$

$$= -\frac{33}{65}$$

(iii) Sin(A –B)

We know that sin(A -B) = sinA cosB - cosA sinB

$= \frac{4}{5} \times \frac{5}{13} - \frac{3}{5} \times \frac{12}{13}$

$= \frac{20}{65} - \frac{36}{65}$

$= \frac{-16}{65}$

(iv) Cos(A –B)

We know that cos(A -B) = cosA cosB + sinA sinB

$= \frac{3}{5} \times \frac{5}{13} + \frac{4}{5} \times \frac{12}{13}$

$= \frac{15}{65} + \frac{48}{65}$

$= \frac{63}{65}$

2 A. Question

If SinA = 12/13 And sinB = 4/5, where π/2<A < π And 0 <B < π/2, find the following:

(i) sin(A +B) (ii) cos(A +B)

Answer

Given sinA = 12/13 And sinB = 4/5 where π/2 <A < π And 0 <B < π/2

We know that $\cos A = -\sqrt{1 - \sin^2 A}$ and $\cos B = \sqrt{1 - \sin^2 B}$

$\Rightarrow \cos A = -\sqrt{1 - \left(\frac{12}{13}\right)^2}$ and $\cos B = \sqrt{1 - \left(\frac{4}{5}\right)^2}$

$\Rightarrow \cos A = -\sqrt{1 - \frac{144}{169}}$ and $\cos B = \sqrt{1 - \frac{16}{25}}$

$\Rightarrow \cos A = -\sqrt{\frac{25}{169}}$ and $\cos B = \sqrt{\frac{9}{25}}$

$\therefore \cos A = -\frac{5}{13}$ and $\cos B = \frac{3}{5}$

(i) Sin(A +B)

We know that sin(A +B) = sinA cosB + cosA sinB

$= \frac{12}{13} \times \frac{3}{5} + \frac{5}{13} \times \frac{4}{5}$

$= \frac{36}{65} - \frac{20}{65}$

$= \frac{16}{65}$

(ii) Cos(A +B)

We know that cos(A +B) = cosA cosB - sinA sinB

$$= -\frac{5}{13} \times \frac{3}{5} - \frac{12}{13} \times \frac{4}{5}$$

$$= -\frac{15}{65} - \frac{48}{65}$$

$$= -\frac{63}{65}$$

2 B. Question

If sinA = 3/5, cosB = –12/13, where A And B Both lie in second quadrant, find the value of sin(A +B).

Answer

Given sinA = 3/5 And cosB = -12/13

A And B lie in the second quadrant.

So sine function is positive And cosine function is negative.

We know that $\cos A = -\sqrt{1-\sin^2 A}$ and $\sin B = \sqrt{1-\cos^2 B}$

$$\Rightarrow \cos A = -\sqrt{1-\left(\frac{3}{5}\right)^2} \text{ and } \sin B = \sqrt{1-\left(\frac{-12}{13}\right)^2}$$

$$\Rightarrow \cos A = -\sqrt{1-\frac{9}{25}} \text{ and } \sin B = \sqrt{1-\frac{144}{169}}$$

$$\Rightarrow \cos A = -\sqrt{\frac{16}{25}} \text{ and } \sin B = \sqrt{\frac{25}{169}}$$

$$\therefore \cos A = -\frac{4}{5} \text{ and } \sin B = \frac{5}{13}$$

Now consider sin(A +B),

$\Rightarrow$ sin(A +B) = $\frac{3}{5} \times \frac{5}{13} + \frac{-4}{5} \times \frac{-12}{13}$

$$= \frac{15}{65} - \frac{48}{65}$$

$$= -\frac{33}{65}$$

3. Question

If cosA = – 24/25 And cosB = 3/5, where π <A < 3π/2 And 3π/2 <B < 2π, find the following:

(i) sin(A +B) (ii) cos(A +B)

Answer

Given cosA = -24/25 And cosB = 3/5 where π <A < 3π/2 And 3π/2 <B < 2π

A is in third quadrant And B is in fourth quadrant.

Here, sine function is negative.

We know that $\sin A = -\sqrt{1-\cos^2 A}$ and $\sin B = -\sqrt{1-\cos^2 B}$

$$\Rightarrow \sin A = -\sqrt{1-\left(-\frac{24}{25}\right)^2} \text{ and } \sin B = -\sqrt{1-\left(\frac{3}{5}\right)^2}$$

$$\Rightarrow \sin A = -\sqrt{1-\frac{576}{625}} \text{ and } \sin B = -\sqrt{1-\frac{9}{25}}$$

$$\Rightarrow \sin A = -\sqrt{\frac{49}{625}} \text{ and } \sin B = -\sqrt{\frac{16}{25}}$$

$$\therefore \sin A = -\frac{7}{25} \text{ and } \sin B = -\frac{4}{5}$$

Then,

(i) Sin(A +B)

We know that sin(A +B) = sinA cosB + cosA sinB

$$= \frac{-7}{25} \times \frac{3}{5} + \frac{-24}{25} \times \frac{-4}{5}$$

$$= \frac{-21}{125} + \frac{96}{125}$$

$$= \frac{75}{125}$$

$$= \frac{3}{5}$$

(ii) Cos(A +B)

We know that cos(A +B) = cosA cosB - sinA sinB

$$= \frac{-24}{25} \times \frac{3}{5} - \frac{-7}{25} \times \frac{-4}{5}$$

$$= \frac{-72}{125} - \frac{28}{125}$$

$$= -\frac{100}{125}$$

$$= -\frac{4}{5}$$

4. Question

If tanA = 3/4, cosB = 9/41, where π<A < 3π/2 And 0 <B < π/2, find tan(A +B).

Answer

Given tanA = 3/4 And cosB = 9/41 where π <A < 3π/2 And 0 <B < π/2

A is in third quadrant And B is in first quadrant.

Tan function And sine function are positive.

We know that $\sin B = \sqrt{1-\cos^2 B}$

$$\Rightarrow \sin B = \sqrt{1-\left(\frac{9}{41}\right)^2}$$

$$= \sqrt{1-\frac{81}{1681}}$$

$= \sqrt{\frac{1600}{1681}}$

$\therefore \sin B = \frac{40}{41}$

We know that $\tan B = \frac{\sin B}{\cos B}$

$= \frac{\frac{40}{41}}{\frac{9}{41}}$

$= \frac{40}{9}$

We know that $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\Rightarrow \tan(A + B) = \frac{\frac{3}{4} + \frac{40}{9}}{1 - \frac{3}{4} \times \frac{40}{9}}$

$= \frac{\frac{187}{36}}{-\frac{84}{36}}$

$= -\frac{187}{84}$

5. Question

If sinA = 1/2, cosB = 12/13, where π/2<A < π And 3π/2 <B < 2π, find tan(A -B).

Answer

Given sinA = 1/2 And cosB = 12/13 where π/2 <A < π And 3π/2 <B < 2π

A is in second quadrant And B is in fourth quadrant.

In the second quadrant, the sine function is positive And cosine And tan functions negative.

In the fourth quadrant, sine And tan functions are negative, And cosine function are positive.

We know that $\cos A = -\sqrt{1 - \sin^2 A}$ and $\sin B = -\sqrt{1 - \cos^2 B}$

$\Rightarrow \cos A = -\sqrt{1 - \left(\frac{1}{2}\right)^2}$ and $\sin B = -\sqrt{1 - \left(\frac{12}{13}\right)^2}$

$\Rightarrow \cos A = -\sqrt{1 - \frac{1}{4}}$ and $\sin B = -\sqrt{1 - \frac{144}{169}}$

$\Rightarrow \cos A = -\sqrt{\frac{3}{4}}$ and $\sin B = -\sqrt{\frac{25}{169}}$

$\therefore \cos A = -\frac{\sqrt{3}}{2}$ and $\sin B = \frac{5}{13}$

$\Rightarrow \tan A = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$ and $\tan B = \frac{-\frac{5}{13}}{\frac{12}{13}} = -\frac{5}{12}$

We know that $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$

$$\Rightarrow \tan(A-B) = \frac{-\frac{1}{\sqrt{3}} - (-\frac{5}{12})}{1+\left(-\frac{1}{\sqrt{3}}\right)\left(-\frac{5}{12}\right)}$$

$$= \frac{\frac{-12+5\sqrt{3}}{12\sqrt{3}}}{\frac{12\sqrt{3}+5}{12\sqrt{3}}}$$

$$= \frac{5\sqrt{3}-12}{5+12\sqrt{3}}$$

6. Question

If SinA = 1/2, cosB = $\frac{\sqrt{3}}{2}$, where π/2<A < π And 0 <B < π/2, find the following:

(i) tan(A +B)(ii) tan(A -B)

Answer

Given sinA = 1/2 And cosB = √3/2 where π/2 <A < π And 0 <B < π/2,

A is in second quadrant And B is in first quadrant.

In the second quadrant, the sine function is positive And cosine And tan functions are negative.

In first quadrant, All functions are positive.

We know that $\cos A = -\sqrt{1-\sin^2 A}$ and $\sin B = \sqrt{1-\cos^2 B}$

$$\Rightarrow \cos A = -\sqrt{1-\left(\frac{1}{2}\right)^2} \text{ and } \sin B = \sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow \cos A = -\sqrt{1-\frac{1}{4}} \text{ and } \sin B = \sqrt{1-\frac{3}{4}}$$

$$\Rightarrow \cos A = -\sqrt{\frac{3}{4}} \text{ and } \sin B = \sqrt{\frac{1}{4}}$$

$$\therefore \cos A = -\frac{\sqrt{3}}{2} \text{ and } \sin B = \frac{1}{2}$$

$$\Rightarrow \tan A = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}} \text{ and } \tan B = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

Then,

(i) tan(A +B)

We know that $\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A \tan B}$

$$\Rightarrow \tan(A+B) = \frac{-\frac{1}{\sqrt{3}} + (\frac{1}{\sqrt{3}})}{1+\left(-\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)}$$

$= \frac{0}{\frac{2}{3}}$

= 0

(ii) tan(A –B)

We know that $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$\Rightarrow \tan(A - B) = \frac{-\frac{1}{\sqrt{3}} - (\frac{1}{\sqrt{3}})}{1 + \left(-\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)}$

$= \frac{-\frac{2}{\sqrt{3}}}{\frac{2}{3}}$

$= -\sqrt{3}$

7. Question

Evaluate the following:

(i) sin 78^0 cos 18^0 – cos 78^0 sin 18^0 (ii) cos 47^0 cos 13^0 - sin 47^0 sin 13^0

(iii) sin 36^0 cos 9^0 + cos 36^0 sin 9^0 (iv) cos 80^0 cos 20^0 + sin 80^0 sin 20^0

Answer

(i) Given sin 78^0 cos 18^0 – cos 78^0 sin 18^0

We know that sin(A -B) = sinA cosB - cosA sinB

⇒ sin 78^0 cos 18^0 – cos 78^0 sin 18^0 = sin(78 – 18) °

= sin 60°

$= \frac{\sqrt{3}}{2}$

$\therefore \sin 78° \cos 18° - \cos 78° \sin 18° = \frac{\sqrt{3}}{2}$

(ii) Given cos 47^0 cos 13^0 - sin 47^0 sin 13^0

We know that cosA cosB – sinA sinB = cos(A +B)

⇒ cos 47^0 cos 13^0 - sin 47^0 sin 13^0 = cos(47 + 13) °

= cos 60°

= 1/2

∴ cos 47^0 cos 13^0 - sin 47^0 sin 13^0 = 1/2

(iii) Given sin 36^0 cos 9^0 + cos 36^0 sin 9^0

We know that sin(A +B) = sinA cosB + cosA sinB

⇒ sin 36^0 cos 9^0 + cos 36^0 sin 9^0 = sin(36 + 9) °

= sin 45°

$= \frac{1}{\sqrt{2}}$

$\therefore \sin 36° \cos 9° + \cos 36° \sin 9° = \frac{1}{\sqrt{2}}$

(iv) Given $\cos 80^0 \cos 20^0 + \sin 80^0 \sin 20^0$

We know that cosA cosB + sinA sinB = cos(A -B)

$\Rightarrow \cos 80^0 \cos 20^0 + \sin 80^0 \sin 20^0 = \cos(80 - 20)°$

$= \cos 60°$

$= 1/2$

$\therefore \cos 80^0 \cos 20^0 + \sin 80^0 \sin 20^0 = 1/2$

8. Question

If cosA = –12/13 and cotB = 24/7, where A lies in the second quadrant and B in the third quadrant, find the values of the following:

(i) sin(A +B) (ii) cos(A +B) (iii) tan(A +B)

Answer

Given cosA = -12/13 And cotB = 24/7

A lies in second quadrant And B in the third quadrant.

The sine function is positive in the second quadrant and in the third quadrant,Both sine And cosine functions are negative.

We know that $\sin A = \sqrt{1 - \cos^2 A}$ and $\sin B = -\frac{1}{\sqrt{1+\cot^2 B}}$

$$\Rightarrow \sin A = \sqrt{1 - \left(-\frac{12}{13}\right)^2} \text{ and } \sin B = -\frac{1}{\sqrt{1 + \left(\frac{24}{7}\right)^2}}$$

$$\Rightarrow \sin A = \sqrt{1 - \frac{144}{169}} \text{ and } \sin B = -\frac{1}{\sqrt{1 + \frac{576}{49}}}$$

$$\Rightarrow \sin A = \sqrt{\frac{25}{169}} \text{ and } \sin B = -\frac{1}{\sqrt{\frac{625}{49}}}$$

$$\therefore \sin A = \frac{5}{13} \text{ and } \sin B = -\frac{7}{25}$$

$$\Rightarrow \cos B = -\sqrt{1 - \sin^2 B} = -\sqrt{1 - \left(-\frac{7}{25}\right)^2} = -\sqrt{1 - \frac{49}{625}} = -\sqrt{\frac{576}{625}} = -\frac{24}{25}$$

Now,

(i) Sin(A +B)

We know that sin(A +B) = sinA cosB + cosA sinB

$$= \frac{5}{13} \times \frac{-24}{25} + \frac{-12}{13} \times \frac{-7}{25}$$

$$= \frac{-120}{325} + \frac{84}{325}$$

$= \frac{-36}{325}$

(ii) Cos(A +B)

We know that cos(A +B) = cosA cosB - sinA sinB

$= \frac{-12}{13} \times \frac{-24}{25} - \frac{5}{13} \times \frac{-7}{25}$

$= \frac{288}{325} + \frac{35}{325}$

$= \frac{323}{325}$

(iii) Tan(A +B)

We know that $\tan(A + B) = \frac{\sin(A+B)}{\cos(A+B)}$

$\Rightarrow \tan(A + B) = \frac{\frac{-36}{325}}{\frac{323}{325}}$

$= -\frac{36}{323}$

9. Question

Prove that: cos 7π/12 + cos π/12 = sin 5π/12 – sin π/12

Answer

⇒ 7π/12 = 105°, π/12 = 15°; 5π/12 = 75°

LHS = cos 105° + cos 15°

= cos(90° + 15°) + sin(90° - 75°)

= -sin 15° + sin 75°

= sin 75° - sin 15° = RHS

Hence proved.

10. Question

Prove that: $\frac{\tan A + \tan B}{\tan A - \tan B} = \frac{\sin(A + B)}{\sin(A - B)}$

Answer

LHS $= \frac{\tan A + \tan B}{\tan A - \tan B}$

$\Rightarrow \frac{\tan A + \tan B}{\tan A - \tan B} = \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{\sin A}{\cos A} - \frac{\sin B}{\cos B}}$

$= \frac{\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B}}{\frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B}}$

We know that sin(A ±B) = sinA cosB ± cosA sinB

$\Rightarrow \frac{\frac{\sin A \cos B+\cos A \sin B}{\cos A \cos B}}{\frac{\sin A \cos B-\cos A \sin B}{\cos A \cos B}} = \frac{\sin(A+B)}{\sin(A-B)}$ = RHS

Hence, proved.

11. Question

Prove that:

(i) $\frac{\cos\ 11^0+\ \sin\ 11^0}{\cos\ 11^0-\ \sin\ 11^0} = \tan 56°$ (ii) $\frac{\cos\ 9^0+\ \sin 9^0}{\cos\ 9^0-\sin\ 9^0} = \tan 54°$

(ii) $\frac{\cos\ 8^0 - \sin\ 8^0}{\cos\ 8^0 +\ \sin\ 8^0} = \tan 37°$

Answer

(i) LHS = $\frac{\cos 11°+\sin 11°}{\cos 11°-\sin 11°}$

Dividing numerator And denominator by cos 11°,

$$\Rightarrow \frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°} = \frac{1 + \tan 11°}{1 - \tan 11°}$$

$$= \frac{1 + \tan 11°}{1 - 1 \times \tan 11°}$$

$$= \frac{\tan 45° + \tan 11°}{1 - \tan 45° \times \tan 11°}$$

We know that $\tan(A + B) = \frac{\tan A+\tan B}{1-\tan A \tan B}$

$$\Rightarrow \frac{\tan 45° + \tan 11°}{1 - \tan 45° \times \tan 11°} = \tan(45° + 11°)$$

= tan 56° = RHS

Hence proved.

(ii) LHS = $\frac{\cos 9°+\sin 9°}{\cos 9°-\sin 9°}$

Dividing numerator And denominator by cos 9°,

$$\Rightarrow \frac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°} = \frac{1 + \tan 9°}{1 - \tan 9°}$$

$$= \frac{1 + \tan 9°}{1 - 1 \times \tan 9°}$$

$$= \frac{\tan 45° + \tan 9°}{1 - \tan 45° \times \tan 9°}$$

We know that $\tan(A + B) = \frac{\tan A+\tan B}{1-\tan A \tan B}$

$$\Rightarrow \frac{\tan 45° + \tan 9°}{1 - \tan 45° \times \tan 9°} = \tan(45° + 9°)$$

= tan 54° = RHS

Hence proved.

(iii) LHS = $\frac{\cos 8°-\sin 8°}{\cos 8°+\sin 8°}$

Dividing numerator And denominator by cos 8°,

$$\Rightarrow \frac{\cos 8° - \sin 8°}{\cos 8° + \sin 8°} = \frac{1 - \tan 8°}{1 + \tan 8°}$$

$$= \frac{1 - \tan 8°}{1 + 1 \times \tan 8°}$$

$$= \frac{\tan 45° - \tan 8°}{1 + \tan 45° \times \tan 8°}$$

We know that $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$$\Rightarrow \frac{\tan 45° - \tan 8°}{1 + \tan 45° \times \tan 8°} = \tan(45° - 8°)$$

= tan 37° = RHS

Hence proved.

12 A. Question

Prove that:

$$\sin\left(\frac{\pi}{3} - x\right)\cos\left(\frac{\pi}{6} + x\right) + \cos\left(\frac{\pi}{3} - x\right)\sin\left(\frac{\pi}{6} + x\right) = 1$$

Answer

We know that sin(A +B) = sinA cosB + cosA sinB

$$\Rightarrow \sin\left(\frac{\pi}{3} - x\right)\cos\left(\frac{\pi}{6} + x\right) + \cos\left(\frac{\pi}{3} - x\right)\sin\left(\frac{\pi}{6} + x\right) = \sin\left(\frac{\pi}{3} - x + \frac{\pi}{6} + x\right)$$

$$= \sin\left(\frac{2\pi + \pi}{6}\right)$$

$$= \sin\left(\frac{\pi}{2}\right)$$

= sin 90°

= 1 = RHS

Hence proved.

12 B. Question

Prove that:

$$\sin\left(\frac{4\pi}{9} + 7\right)\cos\left(\frac{\pi}{9} + 7\right) - \cos\left(\frac{4\pi}{9} + 7\right)\sin\left(\frac{\pi}{9} + 7\right) = \frac{\sqrt{3}}{2}$$

Answer

We know that sin(A -B) = sinA cosB - cosA sinB

$$\Rightarrow \sin\left(\frac{4\pi}{9} + 7\right)\cos\left(\frac{\pi}{9} + 7\right) + \cos\left(\frac{4\pi}{9} + 7\right)\sin\left(\frac{\pi}{9} + 7\right) = \sin\left(\frac{4\pi}{9} + 7 - \frac{\pi}{9} - 7\right)$$

$$= \sin\left(\frac{3\pi}{9}\right)$$

$$= \sin\left(\frac{\pi}{3}\right)$$

= sin 60°

$= \frac{\sqrt{3}}{2}$ = RHS

Hence, proved.

12 C. Question

Prove that:

$$\sin\left(\frac{3\pi}{8}-5\right)\cos\left(\frac{\pi}{8}+5\right)+\cos\left(\frac{3\pi}{8}-5\right)\sin\left(\frac{\pi}{8}+5\right)=1$$

Answer

We know that sin(A +B) = sinA cosB + cosA sinB

$$\Rightarrow \sin\left(\frac{3\pi}{8}-5\right)\cos\left(\frac{\pi}{8}+5\right)+\cos\left(\frac{3\pi}{8}-5\right)\sin\left(\frac{\pi}{8}+5\right) = \sin\left(\frac{3\pi}{8}-5+\frac{\pi}{8}+5\right)$$

$$= \sin(\frac{3\pi+\pi}{8})$$

$$= \sin(\frac{\pi}{2})$$

= sin 90°

= 1 = RHS

Hence proved.

13. Question

Prove that: $\frac{\tan 69^0 + \tan 66^0}{1-\tan 69^0 \tan 66^0} = -1$

Answer

We know that $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

HereA = 69° And B = 66°

$$\text{LHS} = \frac{\tan 69° + \tan 66°}{1 - \tan 69° \tan 66°} = \tan(69+66)°$$

= tan 135°

= - tan 45°

= -1 = RHS

Hence proved.

14 A. Question

If tanA = 5/6 And tanB = 1/11, prove thatA +B = π/4.

Answer

Given $\tan A = \frac{5}{6}; \tan B = \frac{1}{11}$

We know that $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$$\Rightarrow \tan(A+B) = \frac{\frac{5}{6}+\frac{1}{11}}{1-\frac{5}{6}\times\frac{1}{11}}$$

$$= \frac{55+6}{66-5}$$

$= \frac{61}{61}$

= 1

⇒ tan(A +B) = tan π/4

∴A + B = π/4

Hence proved.

14 B. Question

If tanA = m/m–1 And tanB = 1/2m – 1, then prove that A –B = π/4.

Answer

Given $\tan A = \frac{m}{m-1}; \tan B = \frac{1}{2m-1}$

We know that $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$$\Rightarrow \tan(A - B) = \frac{\frac{m}{m-1} - \frac{1}{2m-1}}{1 + \frac{m}{m-1} \times \frac{1}{2m-1}}$$

$$= \frac{2m^2 - m - m + 1}{2m^2 - m - 2m + 1 + m}$$

$$= \frac{2m^2 - 2m + 1}{2m^2 - 2m + 1}$$

= 1

⇒ tan(A –B) = tan π/4

∴A –B = π/4

Hence proved.

15 A. Question

prove that:

$\cos^2 \pi/4 - \sin^2 \frac{\pi}{12} = \frac{\sqrt{3}}{4}$

Answer

LHS = $\cos^2\frac{\pi}{4} - \sin^2\frac{\pi}{12}$

We know that $\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$

$$\Rightarrow \cos^2\frac{\pi}{4} - \sin^2\frac{\pi}{12} = \cos\left(\frac{\pi}{4} + \frac{\pi}{12}\right)\cos\left(\frac{\pi}{4} - \frac{\pi}{12}\right)$$

$$= \cos\frac{4\pi}{12}\cos\frac{2\pi}{12}$$

$$= \cos\frac{\pi}{3}\cos\frac{\pi}{6}$$

$$= \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)$$

$= \frac{\sqrt{3}}{4}$ = RHS

Hence, proved.

15 B. Question

prove that:

$\sin^2(n + 1)A - \sin^2 nA = \sin(2n + 1)A \sin A$

Answer

We know that $\sin^2 A - \sin^2 B = \sin(A + B)\sin(A - B)$

Here $A = (n + 1)A$ And $B = nA$

$\Rightarrow$ LHS: $\sin^2(n + 1)A - \sin^2 nA = \sin((n + 1)A + nA)\sin((n + 1)A - nA)$

$= \sin(nA + A + nA)\sin(nA + A - nA)$

$= \sin(2nA + A)\sin(A)$

$= \sin(2n + 1)A \sin A$ = RHS

Hence proved.

16 A. Question

Prove that:

$$\frac{\sin(A+B)+\sin(A-B)}{\cos(A+B)+\cos(A-B)} = \tan A$$

Answer

LHS $= \frac{\sin(A+B)+\sin(A-B)}{\cos(A+B)+\cos(A-B)}$

We know that $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ And $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\Rightarrow \frac{\sin(A+B)+\sin(A-B)}{\cos(A+B)+\cos(A-B)} = \frac{\sin A\cos B + \cos A\sin B + \sin A\cos B - \cos A\sin B}{\cos A\cos B - \sin A\sin B + \cos A\cos B + \sin A\sin B}$$

$$= \frac{2\sin A\cos B}{2\cos A\cos B}$$

$= \tan A$ = RHS

Hence proved.

16 B. Question

Prove that:

$$\frac{\sin(A-B)}{\cos A\cos B} + \frac{\sin(B-C)}{\cos B\cos C} + \frac{\sin(C-A)}{\cos C\cos A} = 0$$

Answer

LHS $= \frac{\sin(A-B)}{\cos A\cos B} + \frac{\sin(B-C)}{\cos B\cos C} + \frac{\sin(C-A)}{\cos C\cos A}$

We know that $\sin(A - B) = \sin A\cos B - \cos A\sin B$

$$\Rightarrow \frac{\sin(A-B)}{\cos A\cos B} + \frac{\sin(B-C)}{\cos B\cos C} + \frac{\sin(C-A)}{\cos C\cos A} = \frac{\sin A\cos B - \cos A\sin B}{\cos A\cos B} + \frac{\sin B\cos C - \cos B\sin C}{\cos B\cos C} + \frac{\sin C\cos A - \cos C\sin A}{\cos C\cos A}$$

$$= \frac{\sin A \cos B}{\cos A \cos B} - \frac{\cos A \sin B}{\cos A \cos B} + \frac{\sin B \cos C}{\cos B \cos C} - \frac{\cos B \sin C}{\cos B \cos C} + \frac{\sin C \cos A}{\cos C \cos A} - \frac{\cos C \sin A}{\cos C \cos A}$$

= tanA – tanB + tanB – tan C + tan C – tanA

= 0 = RHS

Hence proved.

16 C. Question

Prove that:

$$\frac{\sin(A-B)}{\sin A \sin B} + \frac{\sin(B-C)}{\sin B \sin C} + \frac{\sin(C-A)}{\sin C \sin A} = 0$$

Answer

$$\text{LHS} = \frac{\sin(A-B)}{\sin A \sin B} + \frac{\sin(B-C)}{\sin B \sin C} + \frac{\sin(C-A)}{\sin C \sin A}$$

We know that sin(A –B) = sinA cosB – cosA sinB

$$\Rightarrow \frac{\sin(A-B)}{\sin A \sin B} + \frac{\sin(B-C)}{\sin B \sin C} + \frac{\sin(C-A)}{\sin C \sin A} = \frac{\sin A \cos B - \cos A \sin B}{\sin A \sin B} + \frac{\sin B \cos C - \cos B \sin C}{\sin B \sin C} + \frac{\sin C \cos A - \cos C \sin A}{\sin C \sin A}$$

$$= \frac{\sin A \cos B}{\sin A \sin B} - \frac{\cos A \sin B}{\sin A \sin B} + \frac{\sin B \cos C}{\sin B \sin C} - \frac{\cos B \sin C}{\sin B \sin C} + \frac{\sin C \cos A}{\sin C \sin A} - \frac{\cos C \sin A}{\sin C \sin A}$$

= cotB – cotA + cot C – cotB + cotA – cot C

= 0 = RHS

Hence proved.

16 D. Question

Prove that:

$\sin^2 B = \sin^2 A + \sin^2(A-B) - 2\sin A \cos B \sin(A-B)$

Answer

RHS = $\sin^2 A + \sin^2(A - B) - 2 \sin A \cos B \sin(A - B)$

= $\sin^2 A + \sin(A - B)\,[\sin(A - B) - 2 \sin A \cos B]$

We know that sin(A –B) = sinA cosB – cosA sinB

= $\sin^2 A + \sin(A - B)\,[\sin A \cos B - \cos A \sin B - 2 \sin A \cos B]$

= $\sin^2 A + \sin(A - B)\,[-\sin A \cos B - \cos A \sin B]$

= $\sin^2 A - \sin(A - B)\,[\sin A \cos B + \cos A \sin B]$

We know that sin(A +B) = sinA cosB + cosA sinB

= $\sin^2 A - \sin(A - B) \sin(A + B)$

= $\sin^2 A - \sin^2 A + \sin^2 B$

= $\sin^2 B$ = LHS

Hence proved.

16 E. Question

Prove that:

$\cos^2 A + \cos^2 B - 2 \cos A \cos B \cos(A+B) = \sin^2(A+B)$

Answer

LHS = $\cos^2 A + \cos^2 B - 2 \cos A \cos B \cos(A+B)$

$= \cos^2 A + 1 - \sin^2 B - 2 \cos A \cos B \cos(A+B)$

$= 1 + \cos^2 A - \sin^2 B - 2 \cos A \cos B \cos(A+B)$

We know that $\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$

$= 1 + \cos(A+B)\cos(A-B) - 2 \cos A \cos B \cos(A+B)$

$= 1 + \cos(A+B)[\cos(A-B) - 2 \cos A \cos B]$

We know that $\cos(A-B) = \cos A \cos B + \sin A \sin B$.

$= 1 + \cos(A+B)[\cos A \cos B + \sin A \sin B - 2 \cos A \cos B]$

$= 1 + \cos(A+B)[-\cos A \cos B + \sin A \sin B]$

$= 1 - \cos(A+B)[\cos A \cos B - \sin A \sin B]$

We know that $\cos(A+B) = \cos A \cos B - \sin A \sin B$.

$= 1 - \cos^2(A+B)$

$= \sin^2(A+B)$ = RHS

Hence proved.

16 F. Question

Prove that:

$$\frac{\tan(A+B)}{\cot(A-B)} = \frac{\tan^2 A - \tan^2 B}{1 - \tan^2 A \tan^2 B}$$

Answer

LHS = $\frac{\tan(A+B)}{\cot(A-B)} = \frac{\tan(A+B)}{\frac{1}{\tan(A-B)}}$

We know that $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$$\Rightarrow \frac{\tan(A+B)}{\frac{1}{\tan(A-B)}} = \frac{\frac{\tan A + \tan B}{1 - \tan A \tan B}}{\frac{1}{\frac{\tan A - \tan B}{1 + \tan A \tan B}}}$$

$$= \frac{\tan A + \tan B}{1 - \tan A \tan B} \times \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

We know that $(x+y)(x-y) = x^2 - y^2$

$$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} \times \frac{\tan A - \tan B}{1 + \tan A \tan B} = \frac{\tan^2 A - \tan^2 B}{1 - \tan^2 A \tan^2 B} = \text{RHS}$$

Hence, proved.

17 A. Question

Prove that:

tan 8x – tan6x – tan 2x = tan 8x

tan 6x tan 2x

Answer

We have 8x = 6x + 2x

⇒ tan 8x = tan(6x + 2x)

We know that $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\Rightarrow \tan 8x = \frac{\tan 6x + \tan 2x}{1 - \tan 6x \tan 2x}$

⇒ tan8x (1 – tan6x tan2x) = tan6x + tan2x

⇒ tan8x – tan8x tan 6x tan2x = tan6x + tan2x

∴ tan8x – tan6x – tan2x = tan8x tan6x tan2x

Hence, proved.

17 B. Question

Prove that:

$$\tan\frac{\pi}{12} + \tan\frac{\pi}{6} + \tan\frac{\pi}{12}\tan\frac{\pi}{6} = 1$$

Answer

⇒ π/12 = 15° And π/6 = 30°

We have 15° + 30° = 45°

⇒ tan(15° + 30°) = tan 45°

We know that $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\Rightarrow \frac{\tan 15° + \tan 30°}{1 - \tan 15° \tan 30°} = 1$

⇒ tan15° + tan30° = 1 – tan15° tan30°

∴ tan15° + tan30° + tan15° tan30° = 1

Hence, proved.

17 C. Question

Prove that:

tan 36^0 + tan 9^0 + tan 36^0 tan 9^0 = 1

Answer

We have 36° + 9° = 45°

⇒ tan(36° + 9°) = tan 45°

We know that $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\Rightarrow \frac{\tan 36° + \tan 9°}{1 - \tan 36° \tan 9°} = 1$

⇒ tan 36° + tan 9° = 1 – tan 36° tan 9°

∴ tan 36° + tan 9° + tan 36° tan 9° = 1

Hence proved.

17 D. Question

Prove that:

tan13x – tan 9x – tan 4x = tan 13 x

Tan 9x tan 4x

Answer

We have 13x = 9x + 4x

⇒ tan 13x = tan(9x + 4x)

We know that $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\Rightarrow \tan 13x = \frac{\tan 9x + \tan 4x}{1 - \tan 9x \tan 4x}$

⇒ tan 13x(1 – tan 9x tan 4x) = tan 9x + tan 4x

⇒ tan 13x – tan 13x tan 9x tan 4x = tan 9x + tan 4x

∴ tan 13x – tan 9x – tan 4x = tan 13x tan 9x tan 4x

Hence proved.

18. Question

Prove that: $\frac{\tan^2 2x - \tan^2 x}{1 - \tan^2 2x - \tan^2 x} = \tan 3x \tan x$

Answer

LHS $= \frac{\tan^2 2x - \tan^2 x}{1 - \tan^2 2x \tan^2 x}$

$= \frac{(\tan 2x + \tan x)(\tan 2x - \tan x)}{1 - \tan^2 2x \tan^2 x}$

⇒ tan3x = tan(2x + x) And tan x = tan(2x – x)

$\Rightarrow \frac{(\tan 2x + \tan x)(\tan 2x - \tan x)}{1 - \tan^2 2x \tan^2 x}$

$= \frac{\tan 3x\,(1 - \tan 2x \tan x) \times \tan x\,(1 + \tan 2x \tan x)}{1 - \tan^2 2x \tan^2 x}$

$= \frac{\tan 3x \tan x\,(1 - \tan^2 2x \tan^2 x)}{1 - \tan^2 2x \tan^2 x}$

= tan3x tanx = RHS

Hence, proved.

19. Question

If $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$, show that $\frac{\tan x}{\tan y} = \frac{a}{b}$.

Answer

Given $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$

LHS $= \frac{\sin(A+B) + \sin(A-B)}{\cos(A+B) + \cos(A-B)}$

We know that sin(A ±B) = sinA cosB ± cosA sinB And cos(A ±B) = cosA cosB ∓ sinA sinB

$$\Rightarrow \frac{\sin(A+B)+\sin(A-B)}{\cos(A+B)+\cos(A-B)} = \frac{\sin A\cos B + \cos A\sin B + \sin A\cos B - \cos A\sin B}{\cos A\cos B - \sin A\sin B + \cos A\cos B + \sin A\sin B}$$

$$= \frac{2\sin A\cos B}{2\cos A\cos B}$$

= tanA = RHS

20. Question

If tanA = x tanB, prove that $\frac{\sin(A-B)}{\sin(A+B)} = \frac{x-1}{x+1}$.

Answer

Given tanA = x tanB

LHS = $\frac{\sin(A-B)}{\sin(A+B)}$

We know that sin(A ±B) = sinA cosB ± cosA sinB

$$\Rightarrow \frac{\sin(A-B)}{\sin(A+B)} = \frac{\sin A\cos B - \cos A\sin B}{\sin A\cos B + \cos A\sin B}$$

Dividing numerator And denominator by cosA cosB,

$$\Rightarrow \frac{\sin A\cos B - \cos A\sin B}{\sin A\cos B + \cos A\sin B} = \frac{\tan A - \tan B}{\tan A + \tan B}$$

$$= \frac{x\tan B - \tan B}{x\tan B + \tan B}$$

$$= \frac{(x-1)\tan B}{(x+1)\tan B}$$

$$= \frac{x-1}{x+1}$$

= RHS

Hence, proved.

21. Question

If tan(A +B) = x And tan(A -B) = y, find the values of tan 2A And tan 2B.

Answer

Given tan(A +B) = x And tan(A –B) = y

Consider tan 2A = tan(A +A)

= tan(A +B +A –B)

We know that $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$$\Rightarrow \tan(A+B+A-B) = \frac{\tan(A+B)+\tan(A-B)}{1-\tan(A+B)\tan(A-B)}$$

$$= \frac{x+y}{1-xy}$$

Consider tan 2B = tan(B +B)

= tan(B +A +B –A)

$$= \frac{\tan(B + A) + \tan(B - A)}{1 - \tan(B + A)\tan(B - A)}$$

We know that tan(-θ) = - tan θ

$$= \frac{\tan(A + B) - \tan(A - B)}{1 + \tan(A + B)\tan(A - B)}$$

$$= \frac{x - y}{1 + xy}$$

22. Question

If CosA + SinB = m And SinA + CosB = n, prove that 2 Sin(A +B) = $m^2 + n^2 - 2$.

Answer

Given cosA + sinB = m And sinA + cosB = n

RHS = $m^2 + n^2 - 2$

$=(\cos A + \sin B)^2 + (\sin A + \cos B)^2 - 2$

$= \cos^2 A + \sin^2 B + 2 \cos A \sin B + \sin^2 A + \cos^2 B + 2 \sin A \cos B - 2$

= 1 + 1 + 2(cosA sinB + sinA cosB) - 2

We know that sin(A -B) = sinA cosB - cosA sinB

= 2 sin(A +B)

= LHS

Hence, proved.

23. Question

If tanA + tanB =A And CotA + CotB =B, prove that: cot(A +B) = 1/a – 1/b.

Answer

Given tanA + tanB =A And cotA + cotB =B

Consider cotA + cotB =B

$$\Rightarrow \frac{1}{\tan A} + \frac{1}{\tan B} = b$$

$$\Rightarrow \frac{\tan A + \tan B}{\tan A \ \tan B} = b$$

Then,

RHS $= \frac{1}{a} - \frac{1}{b}$

$$= \frac{1}{\tan A + \tan B} - \frac{1}{\frac{\tan A + \tan B}{\tan A \ \tan B}}$$

$$= \frac{1}{\tan A + \tan B} - \frac{\tan A \tan B}{\tan A + \tan B}$$

$$= \frac{1 - \tan A \tan B}{\tan A + \tan B}$$

We know that $\cot(A + B) = \frac{1 - \tan A \tan B}{\tan A + \tan B}$

= cot(A +B) = LHS

Hence, proved.

24. Question

If x lies in the first quadrant And cos x = 8/17, then prove that

$$\cos\left(\frac{\pi}{6}+x\right)+\cos\left(\frac{\pi}{4}-x\right)+\cos\left(\frac{2\pi}{3}-x\right)=\left(\frac{\sqrt{3}-1}{2}+\frac{1}{\sqrt{2}}\right)\frac{23}{17}$$

Answer

Given x lies in the first quadrant i.e. 0 < x < π/2 And cos x = 8/17

We know that $\sin x = \sqrt{1-\cos^2 x}$

$$\therefore \sin x = \sqrt{1-\frac{64}{289}} = \frac{15}{17}$$

LHS = $\cos\left(\frac{\pi}{6}+x\right)+\cos\left(\frac{\pi}{4}-x\right)+\cos\left(\frac{2\pi}{3}-x\right)$

= cos(30 + x) + cos(45 – x) + cos(120 – x)

We know that cos(A ±B) = cosA cosB ∓ sinA sinB

= cos 30° cos x – sin 30° sin x + cos 45° cos x + sin 45° sin x + cos 120° cos x + sin 120° sin x

= cos x(cos 30° + cos 45° + cos 120°) + sin x(-sin 30° + sin 45° + sin 120°)

$$=\frac{8}{17}\left(\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}-\frac{1}{2}\right)+\frac{15}{17}\left(-\frac{1}{2}+\frac{1}{\sqrt{2}}+\frac{\sqrt{3}}{2}\right)$$

$$=\frac{8}{17}\left(\frac{\sqrt{3}-1}{2}+\frac{1}{\sqrt{2}}\right)+\frac{15}{17}\left(\frac{\sqrt{3}-1}{2}+\frac{1}{\sqrt{2}}\right)$$

$$=\frac{23}{17}\left(\frac{\sqrt{3}-1}{2}+\frac{1}{\sqrt{2}}\right)$$

= RHS

Hence, proved.

25. Question

If $\tan x+\tan\left(x+\frac{\pi}{3}\right)+\tan\left(x+\frac{2\pi}{3}\right)=3$, then prove that $\frac{3\tan x-\tan^3 x}{1-3\tan^2 x}=1$.

Answer

Given $\tan x+\tan\left(x+\frac{\pi}{3}\right)+\tan\left(x+\frac{2\pi}{3}\right) = 3$

We know that $\tan(A + B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$

$$\Rightarrow \tan x+\frac{\tan x+\tan\frac{\pi}{3}}{1-\tan x\tan\frac{\pi}{3}}+\frac{\tan x+\tan\frac{2\pi}{3}}{1-\tan x\tan\frac{2\pi}{3}} = 3$$

$$\Rightarrow \tan x+\frac{\tan x+\sqrt{3}}{1-\sqrt{3}\tan x}+\frac{\tan x-\sqrt{3}}{1+\sqrt{3}\tan x} = 3$$

$$\Rightarrow \frac{\tan x\ (1-3\tan^2 x)+\tan x+\sqrt{3}+\sqrt{3}\tan^2 x+3\tan x+\tan x-\sqrt{3}-\sqrt{3}\tan^2 x+3\tan x}{1-3\tan^2 x}$$

$= 3$

$$\Rightarrow \frac{9\tan x-3\tan^3 x}{1-3\tan^2 x} = 3$$

$$\Rightarrow \frac{3\tan x-\tan^3 x}{1-3\tan^2 x} = 1$$

Hence, proved.

26. Question

If sin(α + β) = 1 And sin(α – β) = 1/2, where $0 \le \alpha, \beta \le \frac{\pi}{2}$, then find the values of tan(α + 2β) And tan(2α + β).

Answer

Given sin(α + β) = 1 And sin(α – β) = 1/2

⇒ α + β = 90° ...(1) And α - β = 30° ...(2)

Adding(1) And(2),

⇒ 2α = 120°

∴ α = 60°

Subtracting(2) from(1),

⇒ 2β = 60°

∴ β = 30°

Then,

∴ tan(α + 2β) = tan(60° + 2 × 30°) = tan 120° = -√3

And tan(2α + β) = tan(2 × 60° + 30°) = tan 150° = -(1/√3)

27. Question

If α,β are two different values of x lying between 0 And 2π which satisfy the equation 6 cos x + 8 sin x = 9, find the value of Sin(α+β).

Answer

Given 6 cos x + 8 sin x = 9

Case 1:

⇒ 6 cos x = 9 – 8 sin x

Squaring on both sides,

⇒ 36 $\cos^2$ x =(9 – 8 sin x)^2

We know that $\cos^2$ x = 1 – $\sin^2$ x.

⇒ 36(1 – $\sin^2$ x) = 81 + 64 $\sin^2$ x – 144 sin x

⇒ 100 $\sin^2$ x – 144 sin x + 45 = 0

∴ cos α And cos β are the roots of the a bove equation

⇒ sin α sin β = 45/100

Case 2:

$\Rightarrow$ 8 sin x = 9 – 6 cos x

Squaring on both sides,

$\Rightarrow$ 64 $\sin^2$ x =(9 – 6 cos x)^2

We know that $\sin^2 x = 1 - \cos^2 x$

$\Rightarrow$ 64(1 – $\cos^2$ x) = 81 + 36 $\cos^2$ x – 108 cos x

$\Rightarrow$ 100 $\cos^2$ x – 108 cos x + 17 = 0

∴ sin α And sin β are the roots of theAbove equation

$\Rightarrow$ cos α cos β = 17/100

Consider cos(α + β),

We know that cos(A +B) = cosA cosB - sinA sinB

$$\Rightarrow \cos(\alpha+\beta) = \frac{17}{100} - \frac{45}{100} = -\frac{28}{100} = -\frac{7}{25}$$

We know that $\sin x = \sqrt{1-\cos^2 x}$

$$\Rightarrow \sin(\alpha+\beta) = \sqrt{1-\cos^2(\alpha+\beta)}$$

$$= \sqrt{1-\left(-\frac{7}{25}\right)^2}$$

$$= \sqrt{\frac{576}{625}}$$

$$= \frac{24}{25}$$

28. Question

If sin α + sin β =A And cos α + cos β =B, show that

(i) $\sin(\alpha+\beta) = \frac{2ab}{a^2+b^2}$

(ii) $\cos(\alpha+\beta) = \frac{b^2-a^2}{b^2+a^2}$

Answer

Given sin α + sin β =A And cos α + cos β =B.

$\Rightarrow A^2 + B^2$ =(sin α + sin β)^2 +(cos α + cos β)^2

= $\sin^2$ α + $\sin^2$ β + 2 sin α sin β + $\cos^2$ α + $\cos^2$ β + 2 cos α cos β

= $\sin^2$ α + $\cos^2$ α + $\sin^2$ β + $\cos^2$ β + 2(sin α sin β + cos α cos β)

We know that cos(A -B) = cosA cosB + sinA sinB

∴$A^2 + B^2$ = 2 + 2 cos(α – β) ...(1)

Then,

$\Rightarrow B^2 - A^2$ =(cos α + cos β)^2 –(sin α + sin β)^2

= $\cos^2$ α + $\cos^2$ β + 2 cos α cos β –($\sin^2$ α + $\sin^2$ β + 2 sin α sin β)

$=(\cos^2 \alpha - \sin^2 \beta) + (\cos^2 \beta - \sin^2 \alpha) - 2\cos(\alpha + \beta)$

$= 2 \cos(\alpha + \beta) \cos(\alpha - \beta) + 2 \cos(\alpha + \beta)$

$= \cos(\alpha + \beta)(2 + 2 \cos(\alpha - \beta))$ ...(2)

From(1) And(2),

$\Rightarrow B^2 - A^2 = \cos(\alpha + \beta)(A^2 + B^2)$

$\therefore \frac{b^2 - a^2}{a^2 + b^2} = \cos(\alpha + \beta)$ ...(ii)

And $\sin(\alpha + \beta) = \sqrt{1 - \cos^2(\alpha + \beta)}$

$$\Rightarrow \sin(\alpha + \beta) = \sqrt{1 - \left(\frac{b^2 - a^2}{b^2 + a^2}\right)^2}$$

$$= \sqrt{\frac{b^4 + a^4 - b^4 - a^4 + 4a^2b^2}{(b^2 + a^2)^2}}$$

$\therefore \sin(\alpha + \beta) = \frac{2ab}{a^2 + b^2}$ ...(i)

29 A. Question

Prove that:

$$\frac{1}{\sin(x - a)\sin(x - b)} = \frac{\cot(x - a) - \cot(x - b)}{\sin(a - b)}$$

Answer

RHS $= \frac{\cot(x-a) - \cot(x-b)}{\sin(a-b)}$

$$= \frac{\frac{\cos(x-a)}{\sin(x-a)} - \frac{\cos(x-b)}{\sin(x-b)}}{\sin(a-b)}$$

$$= \frac{\sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)\sin(a-b)}$$

<u>We know that sin(A -B) = sinA cosB - cosA sinB</u>

$$= \frac{\sin(x - b - x + a)}{\sin(x-a)\sin(x-b)\sin(a-b)}$$

$$= \frac{\sin(a-b)}{\sin(x-a)\sin(x-b)\sin(a-b)}$$

$$= \frac{1}{\sin(x-a)\sin(x-b)}$$

= LHS

Hence, proved.

29 B. Question

Prove that:

$$\frac{1}{\sin(x - a)\cos(x - b)} = \frac{\cot(x - a) + \tan(x - b)}{\cos(a - b)}$$

Answer

RHS $= \frac{\cot(x-a)+\tan(x-b)}{\cos(a-b)}$

$$= \frac{\frac{\cos(x-a)}{\sin(x-a)} + \frac{\sin(x-b)}{\cos(x-b)}}{\cos(a-b)}$$

$$= \frac{\cos(x-b)\cos(x-a) + \sin(x-b)\sin(x-a)}{\sin(x-a)\cos(x-b)\cos(a-b)}$$

We know that cos(A +B) = cosA cosB - sinA sinB

$$= \frac{\cos(x-b-x+a)}{\sin(x-a)\cos(x-b)\cos(a-b)}$$

$$= \frac{\cos(a-b)}{\sin(x-a)\cos(x-b)\cos(a-b)}$$

$$= \frac{1}{\sin(x-a)\cos(x-b)}$$

= LHS

Hence, proved.

29 C. Question

Prove that:

$$\frac{1}{\cos(x-a)\cos(x-b)} = \frac{\tan(x-b)-\tan(x-a)}{\sin(a-b)}$$

Answer

RHS $= \frac{\tan(x-b)-\tan(x-a)}{\sin(a-b)}$

$$= \frac{\frac{\sin(x-b)}{\cos(x-b)} - \frac{\sin(x-a)}{\cos(x-a)}}{\sin(a-b)}$$

$$= \frac{\sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)\sin(a-b)}$$

We know that sin(A -B) = sinA cosB - cosA sinB

$$= \frac{\sin(x-b-x+a)}{\cos(x-a)\cos(x-b)\sin(a-b)}$$

$$= \frac{\sin(a-b)}{\cos(x-a)\cos(x-b)\sin(a-b)}$$

$$= \frac{1}{\cos(x-a)\cos(x-b)}$$

= LHS

Hence, proved.

30. Question

If sin α sin β - cos α cos β + 1 = 0, prove that 1 + cot α tan β = 0.

Answer

Given sin α sin β – cos α cos β + 1 = 0

⇒ -(cos α cos β – sin α sin β) + 1 = 0

We know that cos(A +B) = cosA cosB - sinA sinB

⇒ -cos(α + β) + 1 = 0

⇒ cos(α + β) = 1

We know that sin θ = √(1 – $\cos^2$ θ)

∴ sin(α + β) = 0 ...(1)

Consider 1 + cot α tan β,

$$\Rightarrow 1 + \cot\alpha\tan\beta = 1 + \frac{\cos\alpha\sin\beta}{\sin\alpha\cos\beta}$$

$$= \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\sin\alpha\cos\beta}$$

We know that sin(A ±B) = sinA cosB ± cosA sinB

$$\Rightarrow \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\sin\alpha\cos\beta} = \frac{\sin(\alpha+\beta)}{\sin\alpha\cos\beta}$$

= 0 = RHS

Hence, proved.

31. Question

If tan α = x + 1, tanβ = x - 1, show that 2 cot(α - β) = x^2.

Answer

Given tan α = x + 1 And tan β = x – 1

LHS = 2 cot(α – β)

We know that $\cot(A - B) = \frac{1+\tan A\tan B}{\tan A - \tan B}$

$$\Rightarrow 2\cot(\alpha-\beta) = \frac{2(1+\tan A\tan B)}{\tan A - \tan B}$$

$$= \frac{2 + 2(x+1)(x-1)}{x+1-x+1}$$

$$= \frac{2 + 2x^2 - 2}{2}$$

= x^2 = RHS

Hence, proved.

32. Question

IfAngle θ is divided into two parts such that the tangents of the one part is λ times the tangent of other, And ϕ is their difference, then show that $\sin\theta = \frac{\lambda+1}{\lambda-1}\sin\phi$.

Answer

Let α And β be the two parts of angle θ.

Then, given θ = α + β And ϕ = α - β

Consider tan α = λ tan β

$$\Rightarrow \frac{\tan\alpha}{\tan\beta} = \frac{\lambda}{1}$$

Applying componendo And dividendo,

$$\Rightarrow \frac{\tan\alpha + \tan\beta}{\tan\alpha - \tan\beta} = \frac{\lambda + 1}{\lambda - 1}$$

$$\Rightarrow \frac{\frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta}}{\frac{\sin\alpha}{\cos\alpha} - \frac{\sin\beta}{\cos\beta}} = \frac{\lambda + 1}{\lambda - 1}$$

$$\Rightarrow \frac{\frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta}} = \frac{\lambda + 1}{\lambda - 1}$$

We know that sin(A ±B) = sinA cosB ± cosA sinB

$$\Rightarrow \frac{\sin(\alpha + \beta)}{\sin(\alpha - \beta)} = \frac{\lambda + 1}{\lambda - 1}$$

$$\Rightarrow \frac{\sin\theta}{\sin\phi} = \frac{\lambda + 1}{\lambda - 1}$$

$$\therefore \sin\theta = \frac{\lambda + 1}{\lambda - 1}\sin\phi$$

Hence, proved.

33. Question

If $\tan x = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$, then show that sin α + cos α = $\sqrt{2}$ cos x.

Answer

Given $\tan x = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$

Dividing numerator And denominator on RHSBy cos α,

$$\Rightarrow \tan x = \frac{\tan\alpha - 1}{\tan\alpha + 1}$$

$$= \frac{\tan\alpha - \tan\frac{\pi}{4}}{1 + \tan\alpha\tan\frac{\pi}{4}}$$

We know that $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

$$\Rightarrow \tan x = \tan(\alpha - \frac{\pi}{4})$$

$$\Rightarrow x = \alpha - \frac{\pi}{4}$$

$$\therefore \alpha = \frac{\pi}{4} + x$$

Consider sin α + cos α,

$$\Rightarrow \sin\alpha + \cos\alpha = \sin\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right)$$

We know that sin(A +B) = sinA cosB + cosA sinB And cos(A +B) = cosA cosB - sinA sinB

$\Rightarrow \sin\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right)$

$= \sin\frac{\pi}{4}\cos x + \cos\frac{\pi}{4}\sin x + \cos\frac{\pi}{4}\cos x - \sin\frac{\pi}{4}\sin x$

$= \frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x$

$= \frac{2}{\sqrt{2}}\cos x$

$= \sqrt{2}\cos x$

$\therefore \sin\alpha + \cos\alpha = \sqrt{2}\cos x$

Hence proved.

34. Question

If α And β are two solutions of the equation Atanx + Bsecx = c, then find the values of sin(α + β).

Answer

Given equation Atanx + Bsecx = c

$\Rightarrow$ c – Atanx = Bsecx

Squaring onBoth sides,

$\Rightarrow (c - A\tan x)^2 = (B\sec x)^2$

$\Rightarrow c^2 + A^2\tan^2 x - 2ac\tan x = B^2\sec^2 x$

$\Rightarrow c^2 + A^2\tan^2 x - 2ac\tan x = B^2(1 + \tan^2 x)$

$\Rightarrow (a^2 - B^2)\tan^2 x - 2ac\tan x + (c^2 - B^2) = 0$

There are two solutions tan α And tan β in this quadratic.

$\Rightarrow \tan\alpha + \tan\beta = \frac{2ac}{a^2 - b^2}$ and $\tan\alpha \times \tan\beta = \frac{c^2 - b^2}{a^2 - b^2}$

We know that $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$$\Rightarrow \tan(\alpha + \beta) = \frac{\frac{2ac}{a^2 - b^2}}{1 - \frac{c^2 - b^2}{a^2 - b^2}} = \frac{2ac}{a^2 - c^2}$$

$$\therefore \sin(\alpha + \beta) = \frac{2ac}{a^2 + c^2}$$

Exercise 7.2

1. Question

Find the maximum and minimum values of each of the following trigonometrical expressions:

(i) 12 sin x- 5 cos x

(ii) 12 cos x + 5 sin x+ 4

(iii) $5\cos x + 3\sin\left(\frac{\pi}{6} - x\right) + 4$

(iv) sin x – cos x + 1

Answer

We know that the maximum value of Acosα + Bsinα + c is

$c + \sqrt{(A^2 + B^2)}$

And the minimum value is $c - \sqrt{(a^2 + B^2)}$.

(i) Given f(x) = 12 sin x – 5 cos x

Here A = -5,B = 12 and c = 0

$\Rightarrow -\sqrt{(-5)^2 + 12^2} \le 12\sin x - 5\cos x \le \sqrt{(-5)^2 + 12^2}$

$\Rightarrow -\sqrt{25 + 144} \le 12\sin x - 5\cos x \le \sqrt{25 + 144}$

$\Rightarrow -\sqrt{169} \le 12\sin x - 5\cos x \le \sqrt{169}$

$\Rightarrow -13 \le 12 \sin x - 5 \cos x \le 13$

Hence, the maximum and minimum values of f(x) are 13 and -13 respectively.

(ii) Given f(x) = 12 cos x + 5 sin x + 4

Here A = 12,B = 5 and c = 4

$\Rightarrow 4 - \sqrt{(12)^2 + 5^2} \le 12\cos x + 5\sin x + 4 \le 4 + \sqrt{(12)^2 + 5^2}$

$\Rightarrow 4 - \sqrt{144 + 25} \le 12\cos x + 5\sin x + 4 \le 4 + \sqrt{144 + 25}$

$\Rightarrow 4 - \sqrt{169} \le 12\cos x + 5\sin x + 4 \le 4 + \sqrt{169}$

$\Rightarrow -9 \le 12 \cos x + 5 \sin x + 4 \le 17$

Hence, the maximum And minimum values of f(x) are 17 And -9 respectively.

(iii) Given $f(x) = 5\cos x + 3\sin\left(\frac{\pi}{6} - x\right) + 4$

We know that sin(A -B) = sinA cosB - cosA sinB

$\Rightarrow f(x) = 5\cos x + 3(\sin\frac{\pi}{6}\cos x - \cos\frac{\pi}{6}\sin x) + 4$

$= 5\cos x + \frac{3}{2}\cos x - \frac{3\sqrt{3}}{2}\sin x + 4$

$= \frac{13}{2}\cos x - \frac{3\sqrt{3}}{2}\sin x + 4$

Here $a = \frac{13}{2}; b = -\frac{3\sqrt{3}}{2}; c = 4$

$$\Rightarrow 4 - \sqrt{\left(\frac{13}{2}\right)^2 + \left(-\frac{3\sqrt{3}}{2}\right)^2} \le \frac{13}{2}\cos x - \frac{3\sqrt{3}}{2}\sin x + 4$$
$$\le 4 + \sqrt{\left(\frac{13}{2}\right)^2 + \left(-\frac{3\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow 4 - \sqrt{\frac{169}{4} + \frac{27}{4}} \le \frac{13}{2}\cos x - \frac{3\sqrt{3}}{2}\sin x + 4 \le 4 + \sqrt{\frac{169}{4} + \frac{27}{4}}$$

$$\Rightarrow 4 - 7 \le \frac{13}{2}\cos x - \frac{3\sqrt{3}}{2}\sin x + 4 \le 4 + 7$$

$\Rightarrow -3 \le \frac{13}{2}\cos x - \frac{3\sqrt{3}}{2}\sin x + 4 \le 11$

Hence, the maximum And minimum values of f(x) are 11 And -3 respectively.

(iv) Given f(x) = sin x – cos x + 1

Here A = -1,B = 1 And c = 1

$\Rightarrow 1-\sqrt{(-1)^2+1^2} \le \sin x - \cos x + 1 \le 1+\sqrt{(-1)^2+1^2}$

$\Rightarrow 1-\sqrt{1+1} \le \sin x - \cos x + 1 \le 1+\sqrt{1+1}$

$\Rightarrow 1-\sqrt{2} \le \sin x - \cos x + 1 \le 1+\sqrt{2}$

Hence, the maximum And minimum values of f(x) are $1+\sqrt{2}$ And $1-\sqrt{2}$ respectively.

2 A. Question

Reduce each of the following expressions to the Sine And Cosine of A single expression:

$\sqrt{3}\sin x - \cos x$

Answer

Let f(x) = √3 sin x – cos x

Dividing and multiplying by √(3 + 1) = 2,

$\Rightarrow f(x) = 2(\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x)$

Sine of expression:

$\Rightarrow f(x) = 2(\cos\frac{\pi}{6}\sin x - \sin\frac{\pi}{6}\cos x)$

We know that sinA cosB – cosA sinB = sin(A –B)

$\therefore f(x) = 2\sin(x - \frac{\pi}{6})$

Cosine of the expression:

$\Rightarrow f(x) = 2(\sin\frac{\pi}{3}\sin x - \cos\frac{\pi}{3}\cos x)$

We know that cosA cosB – sinA sinB = cos(A +B)

$\therefore f(x) = -2\cos(\frac{\pi}{3} + x)$

2 B. Question

Reduce each of the following expressions to the Sine And Cosine ofA single expression:

cos x – sin x

Answer

Let f(x) = cos x – sin x

Dividing and multiplying by √(1 + 1) = √2,

$\Rightarrow f(x) = \sqrt{2}(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x)$

Sine of expression:

$\Rightarrow f(x) = \sqrt{2}(\sin\frac{\pi}{4}\cos x - \cos\frac{\pi}{4}\sin x)$

We know that sinA cosB – cosA sinB = sin(A –B)

$\therefore f(x) = \sqrt{2}\sin(\frac{\pi}{4} - x)$

Cosine of the expression:

$\Rightarrow f(x) = 2(\cos\frac{\pi}{4}\cos x - \sin\frac{\pi}{4}\sin x)$

We know that cosA cosB – sinA sinB = cos(A +B)

$\therefore f(x) = \sqrt{2}\cos(\frac{\pi}{4} + x)$

2 C. Question

Reduce each of the following expressions to the Sine And Cosine ofA single expression:

24 cos x + 7 sin x

Answer

Let f(x) = 24 cos x + 7 sin x

Dividing and multiplying by $\sqrt{(24^2 + 7^2)} = \sqrt{625} = 25$,

$\Rightarrow f(x) = 25(\frac{24}{25}\cos x + \frac{7}{25}\sin x)$

Sine of expression:

$\Rightarrow f(x) = 25(\sin\alpha\cos x + \cos\alpha\sin x)$ where sin α = 24/25 And cos α = 7/25

We know that sinA cosB + cosA sinB = sin(A +B)

$\therefore f(x) = 25\sin(\alpha + x)$

Cosine of the expression:

$\Rightarrow f(x) = 25(\cos\alpha\cos x + \sin\alpha\sin x)$

We know that cosA cosB + sinA sinB = cos(A -B)

$\therefore f(x) = 25\cos(\alpha - x)$

3. Question

Show that Sin 100^0 – Sin 10^0 is positive.

Answer

Let f(x) = sin 100° – sin 10°

Dividing And multiplyingBy $\sqrt{(1 + 1)} = \sqrt{2}$,

$\Rightarrow f(x) = \sqrt{2}(\frac{1}{\sqrt{2}}\sin 100° - \frac{1}{\sqrt{2}}\sin 10°)$

$\Rightarrow f(x) = \sqrt{2}(\cos\frac{\pi}{4}\sin(90 + 10)° - \sin\frac{\pi}{4}\sin 10°)$

$\Rightarrow f(x) = \sqrt{2}(\cos\frac{\pi}{4}\cos 10° - \sin\frac{\pi}{4}\sin 10°)$

We know that cosA cosB – sinA sinB = cos(A +B)

$\Rightarrow f(x) = \sqrt{2}\cos(\frac{\pi}{4} + 10°)$

$\therefore f(x) = \sqrt{2}\cos 55°$

4. Question

Prove that $(2\sqrt{3}+3)\sin x + 2\sqrt{3}\cos x$ lies between $-(2\sqrt{3}+\sqrt{15})$ and $(2\sqrt{3}+\sqrt{15})$.

Answer

Let f(x) =(2√3 + 3) sin x + 2√3 cos x

HereA = 2√3 , B = 2√3 + 3 And c = 0

$$\Rightarrow -\sqrt{(2\sqrt{3})^2 + (2\sqrt{3}+3)^2} \le (2\sqrt{3}\ +\ 3)\sin x +\ 2\sqrt{3}\cos x \le \sqrt{(2\sqrt{3})^2 + (2\sqrt{3}+3)^2}$$

$$\Rightarrow -\sqrt{12+12+9+12\sqrt{3}} \le (2\sqrt{3}\ +\ 3)\sin x +\ 2\sqrt{3}\cos x \le \sqrt{12+12+9+12\sqrt{3}}$$

$$\Rightarrow -\sqrt{33+12\sqrt{3}} \le (2\sqrt{3}\ +\ 3)\sin x +\ 2\sqrt{3}\cos x \le \sqrt{33+12\sqrt{3}}$$

Hence proved.

Very Short Answer

1. Question

If α + β - γ = π, and $\sin^2 \alpha + \sin^2 \beta - \sin^2 \gamma = \lambda \sin \alpha \sin \beta \cos \gamma$, then write the value of λ.

Answer

α + β = π + γ

Sin (α + β) =sin (π + γ)

Sin (α) cos (β) + sin (β) cos (α)=-sin(γ)

Take square both side

$[\sin(\alpha)\cos(\beta)+\sin(\beta)\cos(\alpha)]^2=\sin^2(\gamma)$

$\sin^2(\alpha)\cos^2(\beta)+\sin^2(\beta)\cos^2(\alpha)+2\ \text{Sin}(\alpha)\cos(\beta)\sin(\beta)\cos(\alpha)= \sin^2(\gamma)$

$\sin^2(\alpha)[1-\sin^2(\beta)]+\sin^2(\beta)[1-\sin^2(\alpha)]+2\ \text{Sin}(\alpha)\cos(\beta)\sin(\beta)\cos(\alpha)= \sin^2(\gamma)$

$\sin^2(\alpha)-\text{Sin}^2(\alpha)\sin^2(\beta)+\sin^2(\beta)-\sin^2(\beta)\sin^2(\alpha) -\sin^2(\gamma)=- 2\text{Sin}(\alpha)\cos(\beta)\sin(\beta)\cos(\alpha)$

$\sin^2(\alpha)+\sin^2(\beta)-\sin^2(\gamma)=2\text{Sin}^2(\alpha)\sin^2(\beta)- 2\text{Sin}(\alpha)\cos(\beta)\sin(\beta)\cos(\alpha)$

$\sin^2(\alpha)+\sin^2(\beta)-\sin^2(\gamma)=-2\text{Sin}(\alpha)\sin(\beta)[\cos(\beta) \cos(\alpha)- \text{Sin}(\alpha)\sin(\beta)]$

$\sin^2(\alpha)+\sin^2(\beta)-\sin^2(\gamma)=-2\text{Sin}(\alpha)\sin(\beta) \cos(\alpha+ \beta)$

$\sin^2(\alpha)+\sin^2(\beta)-\sin^2(\gamma)=2\text{Sin}(\alpha)\sin(\beta) \sin(\gamma)$

2. Question

If $x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right) = z\cos\left(\theta+\frac{4\pi}{3}\right)$, then write the value of $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$.

Answer

$$x\cos\theta = k = y\cos(\theta+\tfrac{2\pi}{3}) = z\cos(\theta+\tfrac{4\pi}{3})$$

$$z\cos(\theta+\frac{4\pi}{3}) = -z\cos(\theta+\frac{\pi}{3})$$

$\frac{k}{x} = \cos\theta$

$\frac{k}{y} = -\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta$

$\frac{k}{z} = -\left[\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta\right]$

$\frac{k}{x} + \frac{k}{y} + \frac{k}{z} = 0$

3. Question

Write the maximum and minimum values of 3 cos x + 4 sin x + 5.

Answer

the maximum value of (a$\cos x + b\sin x) = \sqrt{a^2 + b^2}$

So maximum value $= \sqrt{9 + 16}$

=5

the minimum value of (a$\cos x + b\sin x) = -\sqrt{a^2 + b^2}$

minimum value= -5

4. Question

Write the maximum values of 12 sin x – 9 $\sin^2$ x.

Answer

$f(x) = 12\sin x - 9(\sin x)^2$

$f(x) = -(3\sin x - 2)^2 + 4$

$-1 \le \sin x \le 1$

$f(x) \in -(3[-1,1]-2)^2+4$

$f(x) \in -([-3,3]-2)^2+4$

$f(x) \in -([-5,1])^2+4$

$f(x) \in -[0,25]+4$

$f(x) \in [-25,0]+4$

$f(x) \in [-21,4]$

5. Question

If 12 sin x – 9 $\sin^2$ x attains its maximum value at x = α, then write the value of sin α.

Answer

$f(x)=12\sin(x) - 9\sin^2(x)$

$f'(x)= 12\cos(x)-18\sin(x)\cos(x)$

$f'(x)=0$ for the maximum value of f(x)

$12\cos(x)-18\sin(x)\cos(x)=0$

$\Rightarrow \sin x = \frac{2}{3}$

6. Question

Write the interval in which the values of $5\cos x + 3\cos\left(x + \frac{\pi}{3}\right) + 3$ lie.

Answer

$f(x) = 5\cos x + 3\cos(x + \frac{\pi}{3}) + 3$

$f(x) = 5\cos x + \frac{3}{2}(\cos x - \sqrt{3}\sin x) + 3$

$f(x) = \frac{13}{2}\cos x - \frac{3}{2}\sqrt{3}\sin x + 3$

$f(x)\epsilon\left[-\sqrt{a^2 + b^2}, \sqrt{a^2 + b^2}\right] + 3$

f(x)ϵ[-7,7]+3

f(x)ϵ[-4,10]

7. Question

If tan (A + B) = p and tan (A – B) = q, then write the value of tan 2B.

Answer

$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

$\tan[(A + B) - (A - B)] = \frac{\tan(A + B) - \tan(A - B)}{1 + \tan(A - B)\tan(A + B)}$

$\tan(2B) = \frac{p - q}{1 + pq}$

8. Question

If $\frac{\cos(x - y)}{\cos(x + y)} = \frac{m}{n}$, then write the value of tan x tan y.

Answer

use componendo and dividendo rule

$\frac{\cos x - y}{\cos x + y} = \frac{m}{n}$

$\frac{\cos(x - y) - \cos(x + y)}{\cos(x - y) + \cos(x + y)} = \frac{m - n}{m + n}$

$\frac{\sin x \sin y}{\cos x \cos y} = \frac{m - n}{m + n}$

$\tan x \tan y = \frac{m - n}{m + n}$

9. Question

If $a = b\cos\frac{2\pi}{3} = c\cos\frac{4\pi}{3}$, then write the value of ab + bc + ca.

Answer

$a = b\cos\frac{2\pi}{3} = c\cos\frac{4\pi}{3} = k$

a=k

b—2k

c=-2k

ab + bc + ca.=0

10. Question

If A + B = C, then write the value of tan A tan B tan C.

Answer

A+B=C

$\tan A + B = \tan C$

tan A + tan B = tan C(1- tan A tan B)

tan C – tan A – tan B = tan A tan B tan C.

11. Question

If sin α – sin β = a and cos α + cos β = b, then write the value of cos (α + β).

Answer

sin α – sin β = a

cos α + cos β = b

(sin α – sin β $)^2$= a^2 1

(cos α + cos $\beta)^2$= b^2 2

add both equations

$2 - 2(\cos \alpha \cos \beta - \sin \alpha \sin \beta) = a^2 + b^2$

value of $\cos (\alpha + \beta) = \frac{a^2+b^2-2}{2}$

12. Question

If $\tan \alpha = \frac{1}{1+2^{-x}}$ and $\tan \beta = \frac{1}{1+2^{x+1}}$, then write the value of α + β lying in the interval (0, π/2).

Answer

Assume x=0

$\tan \alpha = \frac{1}{2}$

$\tan\beta = \frac{1}{3}$

$\tan (\alpha + \beta) = \frac{\tan \alpha + \tan\beta}{1 - \tan \alpha \tan\beta}$

$\tan (\alpha + \beta) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2}\frac{1}{3}}$

$\tan(\alpha + \beta) = \frac{5}{5}$

=1

(α+β) = $\tan^{-1} 1$

$= \frac{\pi}{4}$

MCQ

1. Question

Mark the correct alternative in the following:

The value of i$\sin^2\frac{5\pi}{12} - \sin^2\frac{\pi}{12}$s

A. 1/2

B. √3/2

C. 1

D. 0

Answer

$f(x) = \left(\sin\frac{5\pi}{12}\right)^2 - \left(\sin\frac{\pi}{12}\right)^2$

$f(x) = \sin\left(\frac{5\pi}{12} + \frac{\pi}{12}\right) \sin\left(\frac{5\pi}{12} - \frac{\pi}{12}\right)$

$f(x) = \sin\left(\frac{\pi}{2}\right) \sin\left(\frac{\pi}{3}\right)$

$f(x) = \frac{\sqrt{3}}{2}$

2. Question

Mark the correct alternative in the following:

If A + B + C = π, then sec A (cos B cos C – sin B sin C) is equal to

A. 0

B. -1

C. 1

D. None of these

Answer

B+C= $\pi - A$

TAKE BOTH SIDE COS

Cos (B+C)=cos(π-A)

(cos B cos C – sin B sin C) = -cos(A)

sec A (cos B cos C – sin B sin C) is equal to =-1

3. Question

Mark the correct alternative in the following:

tan 20^o + tan 40^o + √3 tan 20^o tan 40^o is equal to

A. $\frac{\sqrt{3}}{4}$

B. $\frac{\sqrt{3}}{2}$

C. $\sqrt{3}$

D. 1

Answer

tan 20^o + tan 40^o + √3 tan 20^o tan 40^o

tan 60^o (1- tan 20^o tan 40^o)+ √3 tan 20^o tan 40^o

$= \sqrt{3}$

4. Question

Mark the correct alternative in the following:

If $\tan A = \frac{a}{a+1}$ and $\tan B = \frac{1}{2a+1}$, then the value of A + B is

A. 0

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Answer

put a=0

tan(A)=0

tan(B)=1

$\tan A + B = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan A + B = \frac{0 + 1}{1 - 0}$

$\tan A + B = 1$

A+B=$\tan^{-1} 1$

$A + B = \frac{\pi}{4}$

5. Question

Mark the correct alternative in the following:

If 3 sin x + 4 cos x = 5, then 4 sin x – 3 cos x =

A. 0

B. 5

C. 1

D. None of these

Answer

3sin(x)+4cos(x)=5

$\frac{3}{5}\sin x+\frac{4}{5}\cos x=1$

cos(37°-x)=cos 0° (∵ x=37°)

4sin(x)-3cos(x)=k

$4\times\frac{3}{5}-3\times\frac{4}{5}=0$

6. Question

Mark the correct alternative in the following:

If in a ΔABC, tan A + tan B + tan C = 6, then cot A cot B cot C =

A. 6

B. 1

C. 1/6

D. None of these

Answer

A+B=π-C

$\frac{\tan A+\tan B}{1-\tan A\tan B}=-\tan C$

$\tan A+\tan B+\tan C=\tan A\tan B\tan C$

$\cot A\cot B\cot C=\frac{1}{6}$

7. Question

Mark the correct alternative in the following:

tan 3A – tan 2A – tan A is equal to

A. tan 3 A tan 2 A tan A

B. - tan 3 A tan 2 A tan A

C. tan A tan 2 A – tan 2 A tan 3 A – tan 3 A tan A

D. None of these

Answer

tan(A)+tan(B)+tan(C)=tan(A) tan(B)tan(C)

tan 3A – tan 2A – tan A= tan 3 A tan 2 A tan A

8. Question

Mark the correct alternative in the following:

If $A+B+C=\pi$, then $\frac{\tan A+\tan B+\tan C}{\tan A\ \tan B\ \tan C}$ is equal to

A. tan A tan B tan C

B. 0

C. 1

D. None of these

Answer

A+B=π-C

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$$

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C$$

$$\frac{\tan A + \tan B + \tan C}{\tan A \tan B \tan C} = 1$$

9. Question

Mark the correct alternative in the following:

If $\cos P = \frac{1}{7}$ and $\cos Q = \frac{13}{14}$, where P and Q both are acute angles. Then, the value of P – Q is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. $\frac{5\pi}{12}$

Answer

$\cos P = \frac{1}{7}, \sin P = \frac{\sqrt{48}}{7}$

$\cos Q = \frac{13}{14}, \sin Q = \frac{\sqrt{27}}{14}$

Cos(p-q)=cos(p)cos(q)+sin(p)sin(q)

$\cos P - Q = \frac{1}{2}$

$P - Q = \frac{\pi}{3}$

10. Question

Mark the correct alternative in the following:

If cot (α + β) = 0, then sin (α + 2 β) is equal to

A. sin α

B. cos 2 β

C. cos α

D. sin 2 α

Answer

cot (α + β) = 0

$(\alpha + \beta) = \frac{\pi}{2}$

sin (α + 2 β)= sin (α +β) cos(β) + sin(β) cos (α +β)

put $(\alpha + \beta) = \frac{\pi}{2}$

sin (α + 2 β)= cos(β)

11. Question

Mark the correct alternative in the following:

$\frac{\cos 10^\circ + \sin 10^\circ}{\cos 10^\circ - \sin 10^\circ}$ is equal to

A. tan 55^o

B. cot 55^o

C. - tan 35^o

D. - cot 35^o

Answer

$\frac{\cos(10^\circ)+\sin(10^\circ)}{\cos(10^\circ)-\sin(10^\circ)}$

$= \frac{\frac{1}{\sqrt{2}}\cos(10^\circ) + \frac{1}{\sqrt{2}}\sin(10^\circ)}{\frac{1}{\sqrt{2}}\cos(10^\circ) - \frac{1}{\sqrt{2}}\sin(10^\circ)}$

$= \frac{\sin(45^\circ)\cos(10^\circ) + \cos(45^\circ)\sin(10^\circ)}{\cos(45^\circ)\cos(10^\circ) - \sin(45^\circ)\sin(10^\circ)}$

$= \frac{\sin(55^\circ)}{\cos(55^\circ)}$

= tan 55°

12. Question

Mark the correct alternative in the following:

The value of $\cos^2\left(\frac{\pi}{6} + x\right) - \sin^2\left(\frac{\pi}{6} - x\right)$ is

A. $\frac{1}{2}\cos 2x$

B. 0

C. $-\frac{1}{2}\cos 2x$

D. $\frac{1}{2}$

Answer

$\cos^2$ A –$\sin^2$ B

=cos(A+B)cos(A-B)

$= \cos\left(\frac{\pi}{3}\right)\cos(2x)$

$=\frac{1}{2}\cos(2x)$

13. Question

Mark the correct alternative in the following:

If $\tan\theta_1 \tan\theta_2 = k$, then $\frac{\cos(\theta_1-\theta_2)}{\cos(\theta_1+\theta_2)} =$

A. $\frac{1+k}{1-k}$

B. $\frac{1-k}{1+k}$

C. $\frac{k+1}{k-1}$

D. $\frac{k-1}{k+1}$

Answer

$\frac{\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_1}{\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2}$

$=\frac{1+\frac{\sin\theta_1\sin\theta_1}{\cos\theta_1\cos\theta_2}}{1-\frac{\sin\theta_1\sin\theta_1}{\cos\theta_1\cos\theta_2}}$

$=\frac{1+k}{1-k}$

14. Question

Mark the correct alternative in the following:

If sin (π cos x) = cos (π sin x), then sin 2x =

A. $\pm\frac{3}{4}$

B. $\pm\frac{4}{3}$

C. $\pm\frac{1}{3}$

D. None of these

Answer

sin(π cos x)=cos(π sin x)

$\sin\left(\frac{\pi}{2}-\pi\cos x\right) = \cos(\pi\sin x)$

$2n\pi \pm \left(\frac{\pi}{2}-\pi\cos x\right) = (\pi\sin x)$

$\sin x = 2n \pm \left(\frac{1}{2}-\cos x\right)$

Put n=0

$\sin x = \pm(\frac{1}{2} - \cos x)$

$\sin x = (\frac{1}{2} - \cos x)$, $\sin x = (\frac{1}{2} - \cos x)$

$\sin x + \cos x = \frac{1}{2}$, $\sin x - \cos x = -\frac{1}{2}$

Take square both side

$1 + \sin 2x = \frac{1}{4}$, $1 - \sin 2x = \frac{1}{4}$

$\sin 2x = \frac{-3}{4}$, $\sin 2x = \frac{3}{4}$

15. Question

Mark the correct alternative in the following:

If $\tan\theta = \frac{1}{2}$ and $\tan\phi = \frac{1}{3}$, then the value of θ + ϕ is

A. $\frac{\pi}{6}$

B. π

C. 0

D. $\frac{\pi}{4}$

Answer

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan(\theta + \emptyset) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2}\frac{1}{3}}$

$\tan(\theta + \emptyset) = 1$

$(\theta + \emptyset) = \frac{\pi}{4}$

16. Question

Mark the correct alternative in the following:

The value of cos (36° – A) cos (36° + A) + cos (54° + A) cos (54° – A) is

A. sin 2 A

B. cos 2A

C. cos 3A

D. sin 3A

Answer

cos(54° + A) =sin(36° – A)

cos(54° - A) =sin(36° + A)

cos (36°–A) cos (36°+A)+ sin(36°-A)sin(36°+A)=cos(2A)

17. Question

Mark the correct alternative in the following:

If tan (π/4 + x) + tan (π/4 – x) = a, then $\tan^2$ (π /4 + x) + $\tan^2$ (π /4 – x) =

A. $a^2 + 1$

B. $a^2 + 2$

C. $a^2 - 2$

D. None of these

Answer

$$[\tan\left(\frac{\pi}{4}+x\right)]^2 + [\tan\left(\frac{\pi}{4}-x\right)]^2 = \left[\tan\left(\frac{\pi}{4}+x\right) + \tan\left(\frac{\pi}{4}-x\right)\right]^2 - 2\tan\left(\frac{\pi}{4}+x\right)\tan\left(\frac{\pi}{4}-x\right)$$

$$= a^2 - 2\left(\frac{1+\tan(x)}{1-\tan(x)}\right)\left(\frac{1-\tan(x)}{1+\tan(x)}\right)$$

$= a^2 - 2$

18. Question

Mark the correct alternative in the following:

If $\tan(A-B)=1, \sec(A+B)=\frac{2}{\sqrt{3}}$, then the smallest positive value of B is

A. $\frac{25\pi}{24}$

B. $\frac{19\pi}{24}$

C. $\frac{13\pi}{24}$

D. $\frac{11\pi}{24}$

Answer

$A - B = \frac{\pi}{4}$

$A + B = 2\pi - \frac{\pi}{6}$

$B = \frac{19\pi}{24}$

19. Question

Mark the correct alternative in the following:

If A – B = π /4, then (1 + tan A) (1 – tanB) is equal to

A. 2

B. 1

C. 0

D. 3

Answer

$1=\frac{\tan A-\tan B}{1+\tan A \tan B}$

1+tan A tan B=tan A-tan B

tan A-tan B-tan A tan B=1

add both side 1

1+tan A-tan B-tan A tan B=1+1

(1+tan A)(1+tan B)=2

Case2:

put A=0 AND $B=-\frac{\pi}{4}$

(1+tan A)(1-tan A)=1× 2

20. Question

Mark the correct alternative in the following:

The maximum value of $\sin^2\left(\frac{2\pi}{3}+x\right)+\sin^2\left(\frac{2\pi}{3}-x\right)$ is

A. 1/2

B. 3/2

C. 1/4

D. 3/4

Answer

$1-[\cos\left(\frac{2\pi}{3}+x\right)]^2+[\sin(\frac{2\pi}{3}-x)]^2$

1-$\cos\left(\frac{4\pi}{3}\right)\cos(2x)$

$1-\frac{\cos(2x)}{2}$ [$\cos 2x=-1$]

$=\frac{3}{2}$

21. Question

Mark the correct alternative in the following:

If $\cos(A-B)=\frac{3}{5}$ and tan A tan B = 2, then

A. $\cos A \cos B=\frac{1}{5}$

B. $\cos A \cos B=-\frac{1}{5}$

C. $\sin A \sin B=-\frac{1}{5}$

D. $\sin A \sin B = \frac{1}{5}$

Answer

Cos(A-B) =cos(A)cos(B)+sin(A)sin(B)

$\frac{3}{5} = \cos A \cos B + \sin A \sin B$

$\frac{3}{5 \cos A \cos B} = 1 + \frac{\sin A \sin B}{\cos A \cos B}$

$\frac{3}{5 \cos A \cos B} = 1 + 2$

$\frac{1}{5} = \cos A \cos B$

22. Question

Mark the correct alternative in the following:

If tan 69^o + tan 66^o – tan 69^o tan 66^o = 2k, then k =

A. -1

B. 1/2

C. -1/2

D. None of these

Answer

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan(69° + 66°) = \frac{\tan 69° + \tan 66°}{1 - \tan 69° \tan 66°}$

$-1 = \frac{\tan 69° + \tan 66°}{1 - \tan 69° \tan 66°}$

-1+tan 69° tan66°=tan 69°+tan66°

tan 69°+tan66°-tan 69° tan66°=-1

2k=-1

$k = -\frac{1}{2}$

23. Question

Mark the correct alternative in the following:

If $\tan \alpha = \frac{x}{x+1}$ and $\tan \beta = \frac{1}{2x+1}$, then α + β is equal to

A. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{4}$

Answer

put x=1

$\tan\alpha = \frac{1}{2}$

$\tan\beta = \frac{1}{3}$

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\tan(\alpha+\beta) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2}\frac{1}{3}}$

tan(α+β)=1

$(\alpha+\beta) = \frac{\pi}{4}$